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Curves, Moduli and Integrable Systems

Date: February 17-19, 2015

Place: room 5206, Building 5, Tsuda College

Address: Tsuda-machi 2-1-1, Kodaira 187-8577, Tokyo, Japan

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February 17 (Tuesday)

9:45-10:45 **Hironori Shiga**

Hypergeometric modular functions and the complex multiplication

11:00-12:00 **Tomohide Terasoma** (University of Tokyo)

Period map of mixed Hodge structures for certain triple coverings

12:15-13:15 **Yoshihiro Onishi** (Meijo University)

New addition formulae for Weierstrass elliptic functions and for higher genus Abelian functions

15:00-16:00 **Kenji Koike** (Yamanashi University)

Cyclic heptagonal curves and hypergeometric periods

16:30-17:30 **Atsushi Nakayashiki** (Tsuda College)

On the series expansions of higher genus sigma functions

February 18 (Wednesday)

9:45-10:45 **Dmitry Korotkin** (Concordia University)

Poisson geometry of Schrodinger equation on Riemann surfaces

11:00-12:00 **Yasuhiko Yamada** (Kobe University)

Quantum curves associated with quantum Painlevé equations

12:15-13:15 **Oleg Chalykh** (University of Leeds)

KP hierarchy for a cyclic quiver

15:00-16:00 **Kanehisa Takasaki** (Kinki University)

Integrable structure of various melting crystal models

16:30-17:30 **Rei Inoue** (Chiba University)

Toric networks and generalizations of disctete Toda lattice

February 19 (Thursday)

9:45-10:45 **Harry Braden** (University of Edinburgh)

Monopoles and Moduli

11:00-12:00 **Victor Enolski** (Tsuda College and Institute of Magnetism, Natl. Acad. Sci. Ukraine)

Aspects of Hitchin's monopole calculations within the Nahm Ansatz

12:15-13:15 **Andrey Mironov** (Sobolev Institute)

Commuting ordinary differential operators with polynomial coefficients and automorphisms of the first Weyl algebra

15:00-16:00 **Takahiro Shiota** (Kyoto University)

Soliton equations and Prym varieties

Organizing committee

Victor Enolski (Tsuda College and Institute of Magnetism, NASU)
Atsushi Nakayashiki (Tsuda College)
Teruhisa Tsuda (Hitotsubashi University)

Abstracts

Harry Braden

The modern approach to integrability proceeds via a Riemann surface, the spectral curve. In many applications this curve is specified by transcendental constraints in terms of periods. I will highlight some of the problems this leads to in the context of monopoles giving new results.

Oleg Chalykh

Motivated by the bispectral problem, we introduce a version of the KP hierarchy, related to the cyclic quiver with m vertices. The case m=1 corresponds to the usual KP hierarchy. We construct families of rational solutions to this hierarchy, parameterised by suitable quiver varieties. The pole dynamics of these solutions is related to the Calogero-Moser system for the wreath product of S_n and Z/mZ. This is joint work with A Silantyev.

Victor Enolski

The ADHMN-construction (Atiyah-Drinfeld-Hitchin-Manin-Nahm) of the Higgs field and gauge fields for a nonabelian monopole reduces to solving a Weyl equation, a linear ODE with "potentials" given by the so called Nahm data. Even in the case of charge two, where the Nahm data is expressible in terms of elliptic functions, the analytical expressions for the monopole fields are still unknown. We have solved this problem using another ansatz of Nahm, less well known compared with the ADHMN construction: the implementation of this ansatz leads to intrinstic theta-functional relations. The exposition of the and allied problems will be the subject of the talk.

Rei Inoue

We define generalizations of discrete Toda lattice, by using the networks on a torus. Each such system has a family of commuting rational maps, which comes from the extended symmetric group action on the network. We describe the maps in terms of algebro-geometrical data, and study the initial value ploblem. It is based on a joint work with Thomas Lam and Pavlo Pylyavskyy.

Kenji Koike

I will introduce the period map via hypergeometric functions for a 1-dimensional family of cyclic 7-gonal curves, and related Tomae type formula. The moduli space of Jacobians for this family (with a certain level structure) is isomorphic to the Fermat curve of degree 7.

Dmitry Korotkin

The potential of the Schrodinger equation on a Riemann surface can always be represented as sum of Bergman projective connection and a holomorphic quadratic differential. The quadratic differential determines a canonical two-sheeted covering of the Riemann surface, and the Poisson structure is defined via homological coordinates

defined by this covering. Starting from this Poisson structure we compute Poisson brackets between monodromy matrices which turns out to give the Goldman bracket. Being combined with results of Kawai this proves the equivalence with canonical Poisson bracket on cotangent bundle to the moduli space of Riemann surfaces. We discuss also the Poisson structure of Voros symbols which arise in WKB approximation. The talk is based on joint work with M.Bertola and C.Norton

Andrey Mironov

We discuss new results on commuting ordinary differential operators of rank two. In particular we point out an connection between eigenfunctions of one-dimensional Schrodinger operator with polynomial potentials of degree 3, 4 and eigenfunctions of rank two commuting ordinary differential operators. We also study an action of the group of automorphisms of the first Weyl algebra on commuting ordinary differential operators with polynomial coefficients. We show that in the case of spectral curves of genus one the space of orbits is infinite. The results are obtained with B.T.Saparbaeva and A.B.Zheglov.

Atsushi Nakayashiki

By studying the properties of the tau function of an integrable hierarchy of soliton equations and Schur functions we derive results on various series expansions of the sigma functions of a Riemann surface defined by Korotkin and Shramchenko.

Yoshihiro Onishi

For the Weierstrass functions $\sigma(u)$and $\wp(u)$ we know the formula $\sigma(u+v)\sigma(u-v)/\sigma(u)^2\sigma(v)^2 = \wp(v) - \wp(u)$ and three or more variable generalization of this. This formula corresponds to canonical involution $v \mapsto -v$. Through my recent researches, I realized that this involution should be regarded as the exchange of the two points v and $-v$ which appear as the inverse image of the function $v \mapsto \wp(v)$. This means that it is natural to ask what happens if we replace the function $\wp(u)$ by another function. Investigating from this point of view and treating a function of any order, we get some new addition formulae. I will talk about higher genus case also from this point of view. This is a joint work with J.C. Eilbeck and M. England.

Hironori Shiga

We consider the inverse of the Schwarz map for the hypergeometric differential equation (one or several varables). In some cases it becomes to be a modular function for a special family of algebraic varieties, and also we get its representation by theta constants. We discuss about the arithmetic property of such modular functions from the view point of the theory of complex multiplication by Shimura.

Takahiro Shiota

to be announced

Kanehisa Takasaki

I will review my recent work on integrable structure of melting crystal models. The simplest model is a statistical model of random 3D Young diagrams. A variant of this model originates in topological string theory on the resolved conifold. These two models are known to be related to the 1D Toda hierarchy and the Ablowitz-Ladik hierarchy, respectively. I found that these results can be extended to "orbifold" models. The relevant integrable systems are particular reductions of the 2D Toda hierarchy.

Tomohide Terasoma

Let X be a triple covering of the projective plane branching at 6 lines. If 6 lines has two triple points, then X becomes K3 a surface with singularities and the cohomology of X has mixed Hodge structure. In this case, the corresponding period domain is the product of the upperhalf plane and the two dimensional complex vector space.

Using genus two theta functions, we construct concrete inverse period map. It is remarkable that one can not recover the variety X without considering the extension class for mixed Hodge structures.

This is joint work with Matsumoto, Sasaki, Yoshida.

Yasuhiko Yamada

The tau-functions (or the tau-cocycles) of the Painlevé equations have a geometric characterization as algebraic curves. This is closely related with the Takano's geometric characterization of the Hamiltonians of the Painlevé equations. In this talk, I will show similar characterization for the quantum analogue of the tau-functions which were introduced by G.Kuroki in 2012.