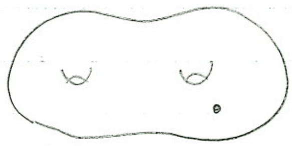
11:15 - 12:05 M. Bell
"Flip graphs and encodings of mapping classes"

S: surface genus g w/ $n \geq 0$ punctures



$\circlearrowleft h$ (up to isotopy)

$g = 2$
$n = 1$

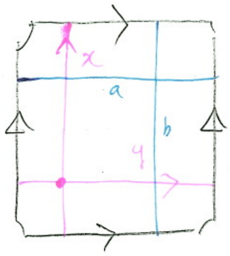
Fundamental Group

loops on S

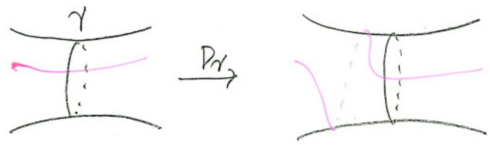
Data structures for mapping classes and loops and nice action.

$\pi_1(S) \cong F_k$
loops $\leftrightarrow$ word in F_k
mapping class $\leftrightarrow$ isomorphism of F_k

Example



$\pi_1(S) = \langle x, y \rangle$



$D_a: x \mapsto xy, \ y \mapsto y$
$D_b^{-1}: x \mapsto x, \ y \mapsto yx$

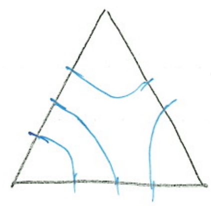
$$\begin{matrix} {}^{1}x \\ {}^{1}y \end{matrix} \xmapsto{D_b^{-1}D_a} \begin{matrix} {}^{3}xyx \\ {}^{2}yx \end{matrix} \xmapsto{D_b^{-1}D_a} \begin{matrix} {}^{8}xyxyxxyx \\ {}^{5}yxxyx \end{matrix}$$

$$\xmapsto{D_b^{-1}D_a} \begin{matrix} {}^{21}xyxyxxyxyxxyxxyxyxxyx \\ {}^{13}yxxyxxyxyxxyx \end{matrix}$$

Trianglations

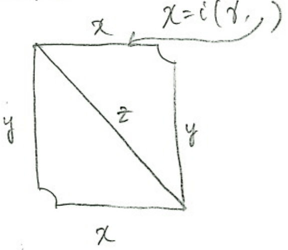
triangulation $\mathcal{T}$ of S

$$\underset{\text{s.c.c.}}{\gamma} \leftrightarrow \begin{pmatrix} i(\gamma, e_1) \\ \vdots \\ i(\gamma, e_k) \end{pmatrix} = \mathcal{T}(\gamma) \in \mathbb{N}^k$$



mapping classes $\leftrightarrow$ PL-functions on these vectors

Example



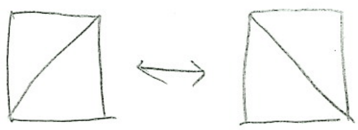
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longmapsto \begin{pmatrix} \max(2x-y,\ 2z-y) \\ z \\ \max(2z-x,\ x-y,\ 2z-x-y) \end{pmatrix}$$

Flip Graph

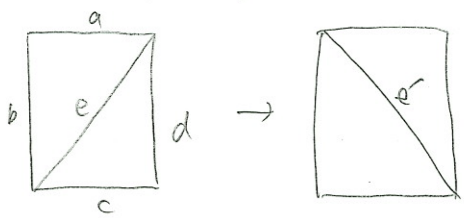
$G = G(S)$: graph

$\Leftrightarrow$ { vertices $\leftrightarrow$ triangulation

$T - T'$ iff they differ by a "flip"



h can be rep. via path from T to $h(T)$.



$e' = \max(a+c,\ b+d) - e$

How do you find such a path?

$g = 2,\ n = 1$

$\mathcal{M}(S) = G(S) / MCG(S)$

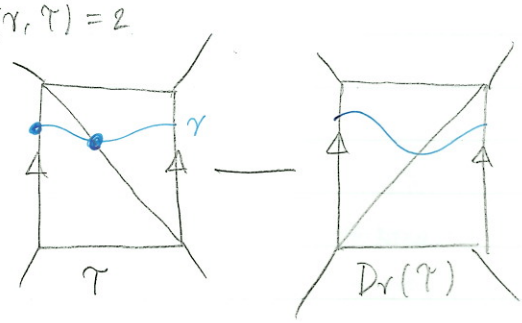
Dehn twists

given γ can compute a path

$T - \cdots - D_\gamma(T)$ efficiently locally

Simple case

$i(\gamma, T) = 2$

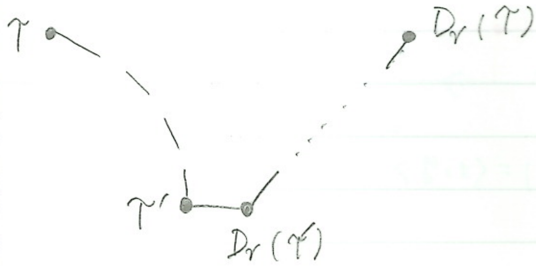


Lem.

if $i(\gamma, T) > 2$ (and γ is non-sep. curve)

then $T - T' - T''$ s.t.

$i(\gamma, T'') < i(\gamma, T)$



15:05 - 16:05 M. Bell

"The conjugacy problem for mapping class groups"

<u>Conjugacy Problem</u>

group $G = \langle X \rangle$ ← finite

Given $g, h \in G$ decide if $\exists w$ s.t.

$g = w h w^{-1}$

$\boxed{\text{Undecidable}}$

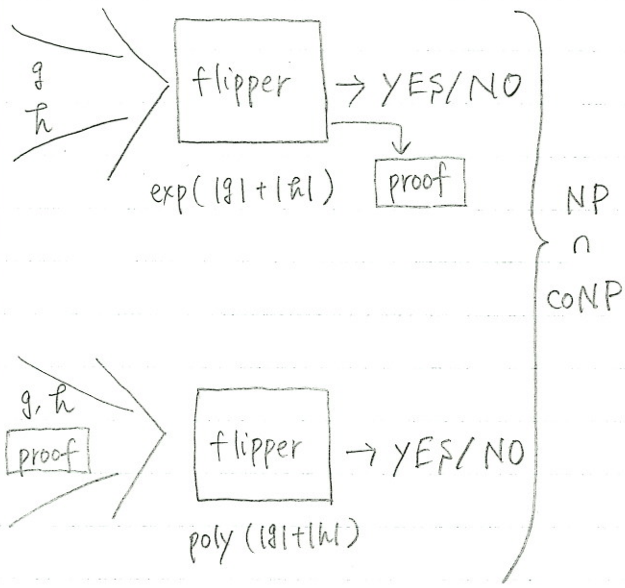
<u>Theorem</u> [B.]

When $G = MCG(S)$ $g>0, n>0$,

this problem is decidable and

lives in $NP \cap coNP$

aB, abaBBB



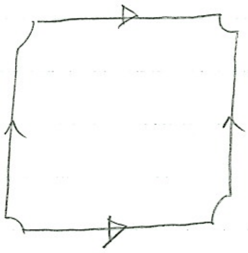
No. 18

Date

Thm. [Jing Tao, Oaklahoma]
if g is conjugate to h then $\exists$ a conjugator w
$g = w h w^{-1}$ and $|w| \le K(|g| + |h|)$
$K = K(S, X)$

Thm. [Mosher]
$MCG(S)$ has a poly. time solution to the word problem.

$\mathbb{R}^2 - \mathbb{Z}^2$



$\circlearrowleft h \leftrightarrow \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

eigenvector : $\begin{pmatrix} \varphi \\ 1 \end{pmatrix} \leftarrow \frac{1+\sqrt{5}}{2}$

continued fraction expansion :

$$\varphi := 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}} = [1; \underline{1, 1, 1}, \cdots]$$

$$\begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}^{-1}$$

eigenvector

$$\begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} \varphi \\ 1 \end{pmatrix} \equiv \begin{pmatrix} \frac{9-\sqrt{5}}{2} \\ 1 \end{pmatrix}$$

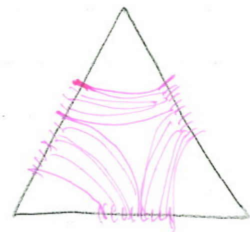
$$\frac{9-\sqrt{5}}{2} = [3; 2, \underline{1, 1, 1}, \cdots]$$

the periodic part is a total conjugacy invariant.

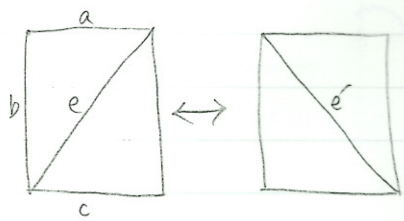
__Assume__

h is pseudo-Anosov - $\begin{cases} \infty \text{ order} \\ \text{irreducible} \end{cases}$

Theorem
h has a projectively invariant measured lamination.



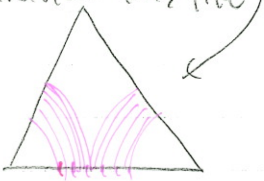
repeat constructions from Lecture 1 w/ laminations.



$e' = \max(a+c, b+d) - e$

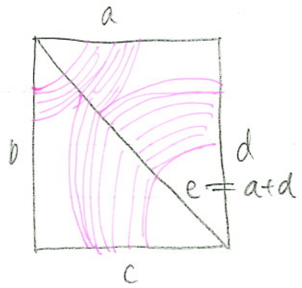
<u>Assume</u>

lamination looks like

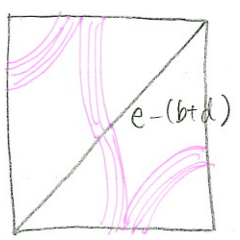


$$\begin{pmatrix} 3.38 \\ 1 \end{pmatrix} \to \begin{pmatrix} 2.38 \\ 1 \end{pmatrix} \to \begin{pmatrix} 1.38 \\ 1 \end{pmatrix} \to \begin{pmatrix} 0.38 \\ 1 \end{pmatrix}$$

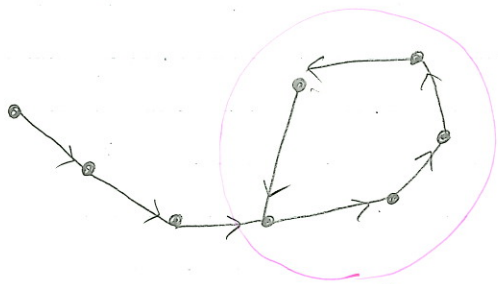
$$\to \begin{pmatrix} 0.38 \\ 0.62 \end{pmatrix} \to \begin{pmatrix} 0.38 \\ 0.24 \end{pmatrix} \to$$



$\downarrow$ flip



Thm. [Agol]
The sequence of measures is eventually periodic.
(up to rescaling, reembedding)



$$\tau(\alpha) = \begin{pmatrix} \alpha(e_1) \\ \vdots \\ \alpha(e_k) \end{pmatrix} \in \mathbb{R}^k$$

a total conjugacy invariant

Thm. [B]
"eventually" $= O(|h|^2)$