Warping Incidence Matrix & Reidemeister Moves

by Taira Akiyama & Ayaka Shimizu
National Institute of Technology, Gunma College

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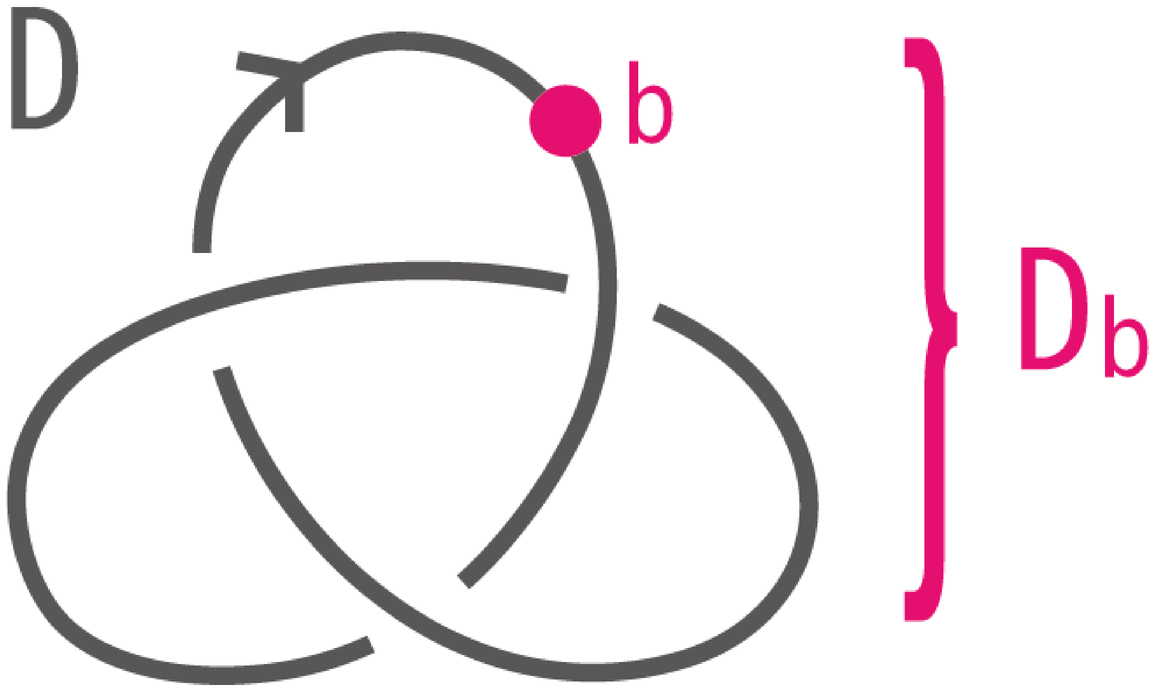
§1. Warping Incidence Matrix

Warping Incidence Matrix

D: an oriented knot diagram on S^2

b: a base point of D



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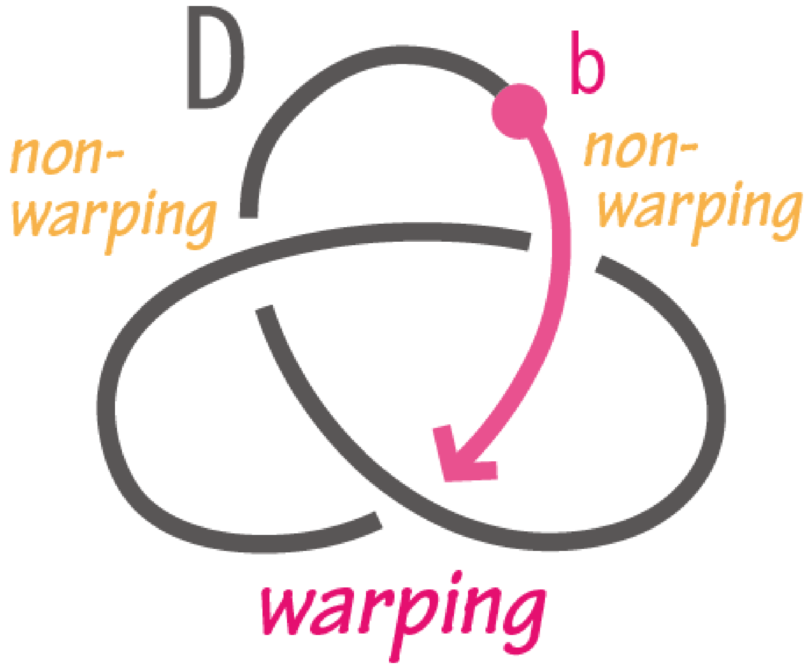
Warping Incidence Matrix

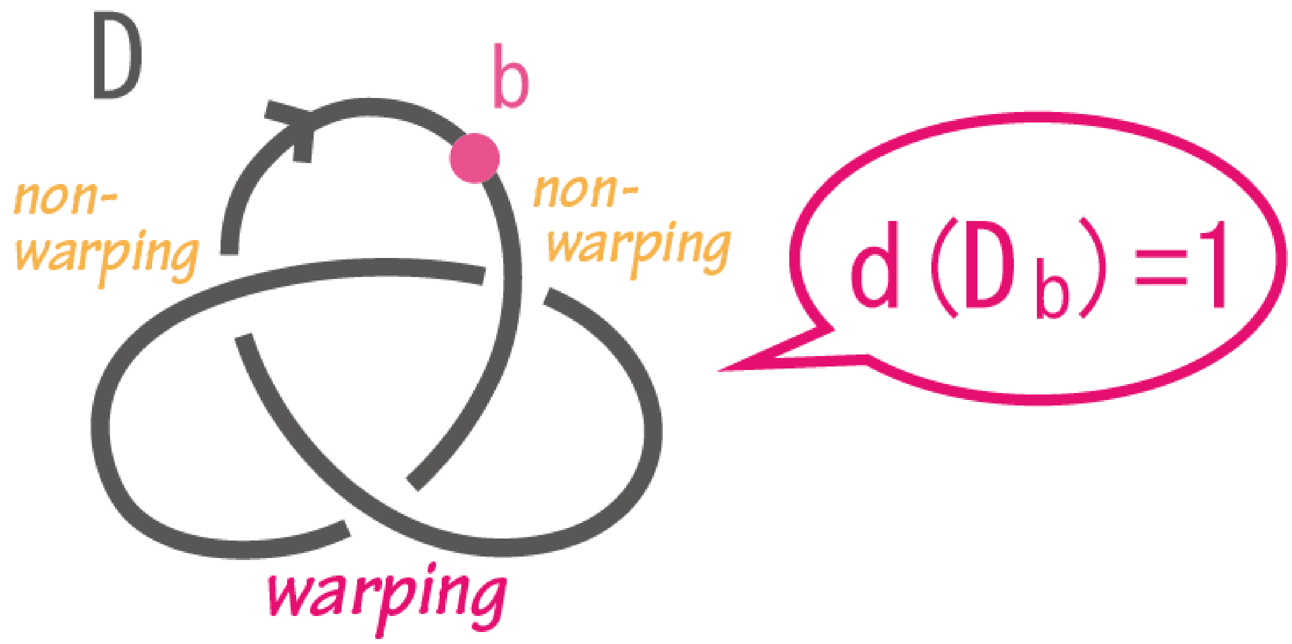
A crossing point p of D is a warping crossing point of D_b if we meet p as an undercrossing first when we travel D from b.



Warping Incidence Matrix

The **warping degree** $d(D_b)$ of D_b is the number of the warping crossing points of D_b.

Example



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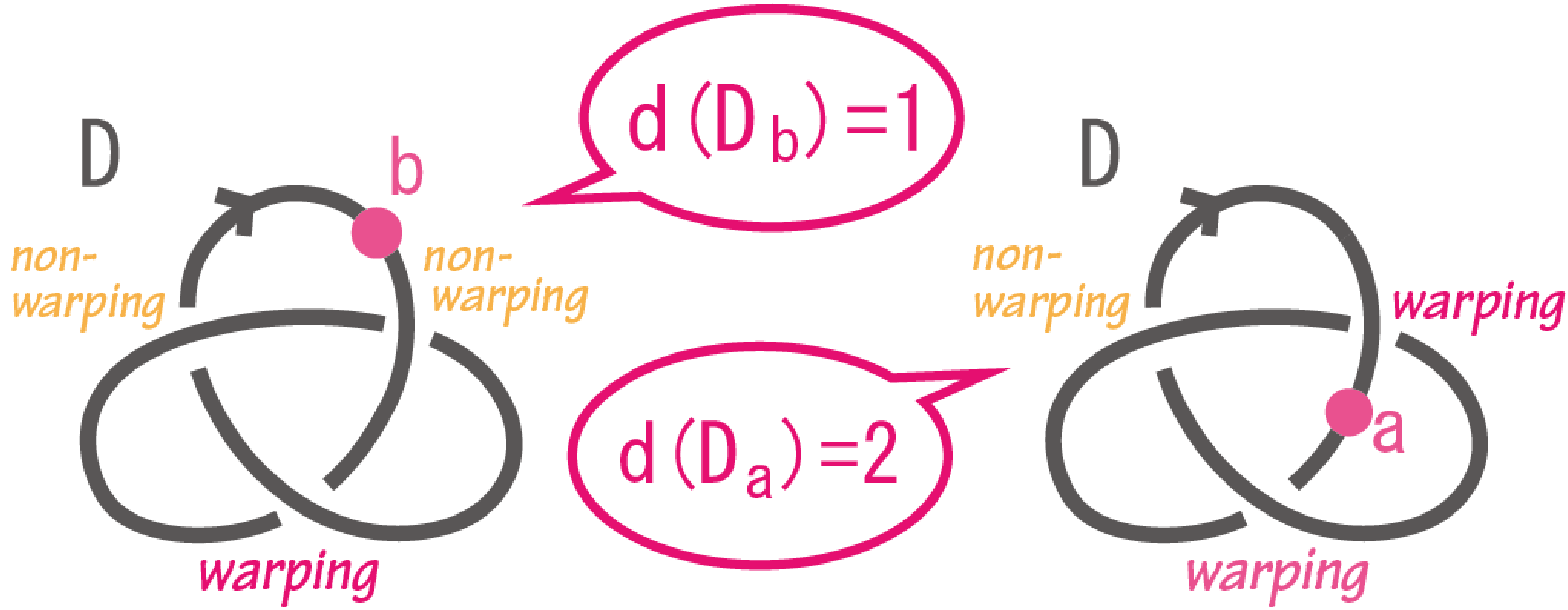
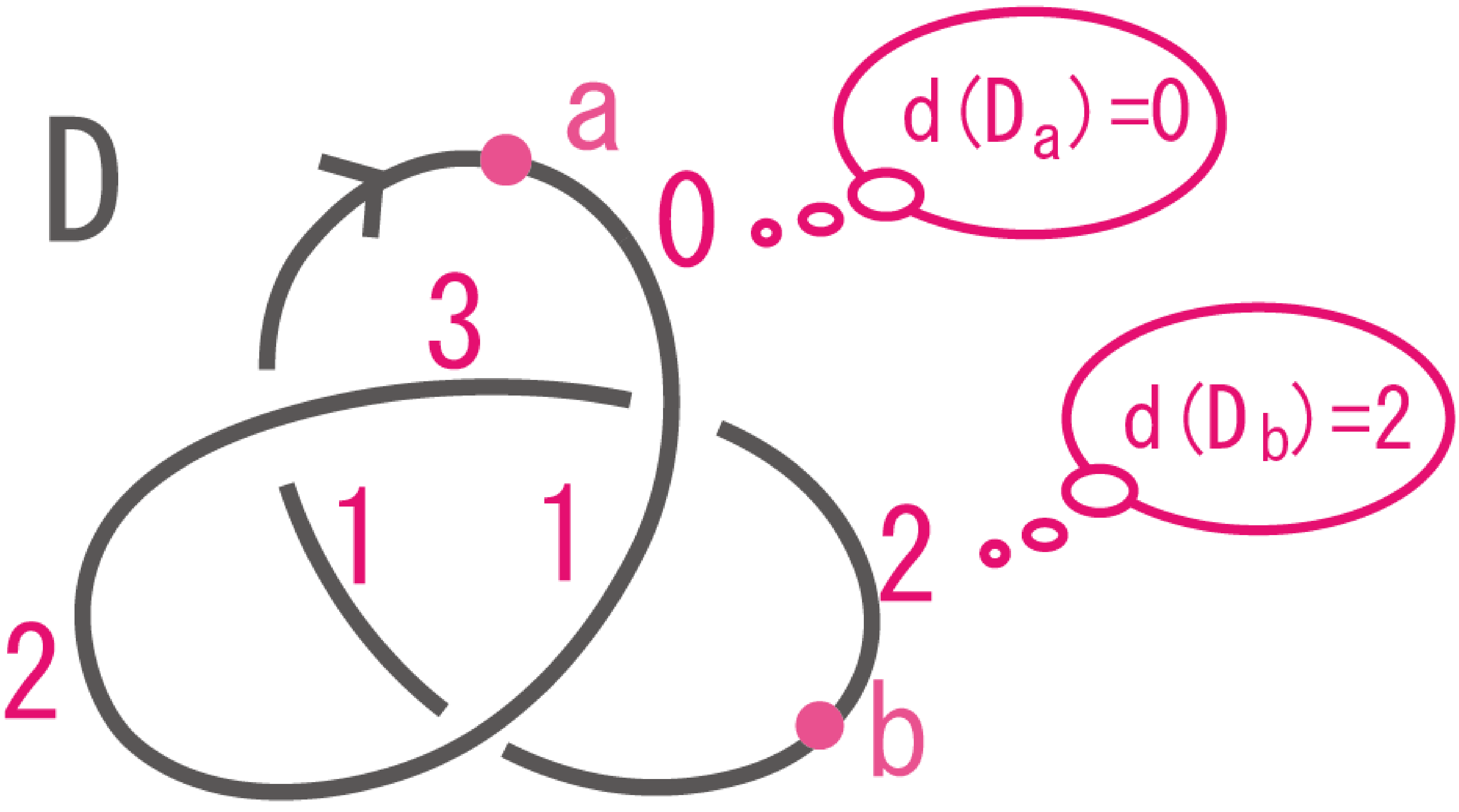
Remark

Warping degree depends on the choice of base point.



Warping Incidence Matrix

Warping degree labeling



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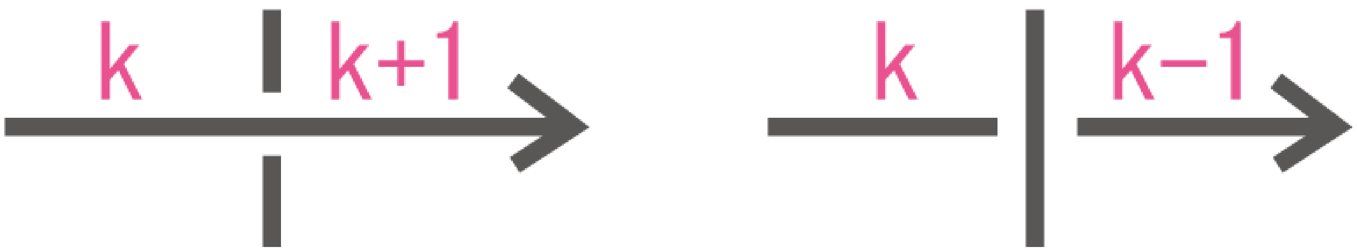
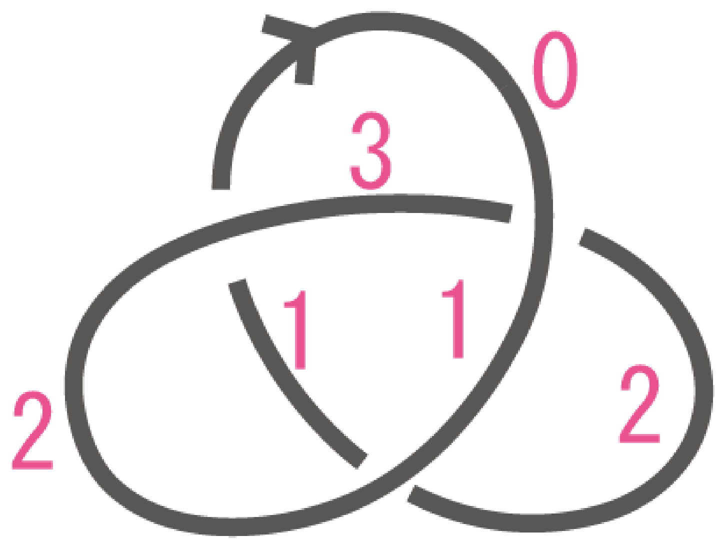
Warping Incidence Matrix

Property





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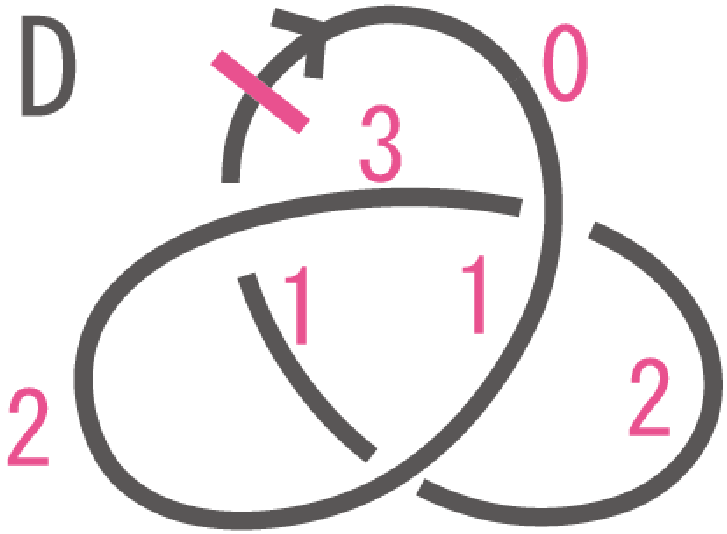
Warping Incidence Matrix



Warping degree sequence

012321 / cyclic permutation

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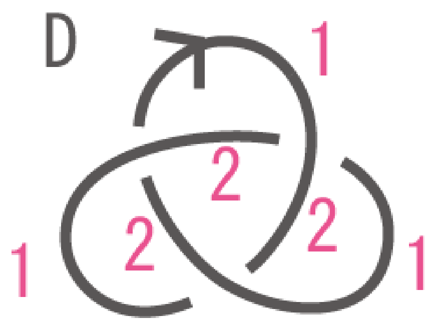
Warping Incidence Matrix

warping degree sequence



121212

warping degree

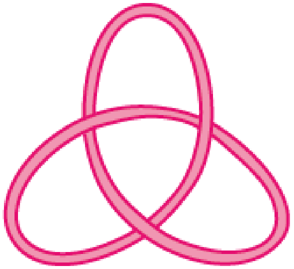
$d(D)=1$

warping polynomial

$W_D(t)=3t^2+3t$

warping matrix

$$\begin{pmatrix} 0 & 1 & \bar{2} & 3 & 2 & 1 \\ 1 & 0 & \bar{1} & 2 & \bar{3} & 2 \\ 2 & 1 & 0 & 1 & \bar{2} & 3 \\ \bar{1} & 2 & \bar{3} & 2 & 1 & 0 \\ \bar{2} & 1 & \bar{2} & 1 & \bar{2} & 1 \\ \bar{2} & 3 & 2 & 1 & 0 & 1 \\ \bar{3} & 2 & 1 & 0 & \bar{1} & 2 \end{pmatrix} \mapsto$$



Q Reidemeister move?

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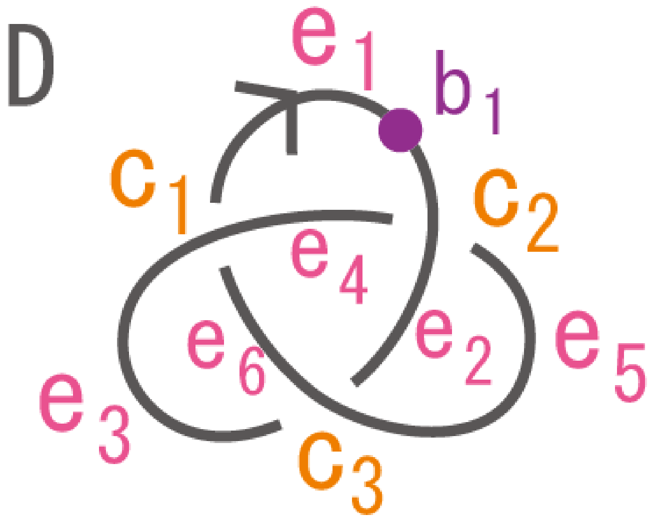
Warping Incidence Matrix



$e_1, e_2, \ldots e_{2n}$:edges

$c_1, c_2, \ldots c_n$:crossings

warping incidence matrix $m(D) = (m_{ij})$

$$m_{ij} = \begin{cases} 1 & (c_i \text{ is a warping crossing point of } D_{b_j}) \\ 0 & (\text{otherwise}) \end{cases}$$

b_j is a base point on e_j

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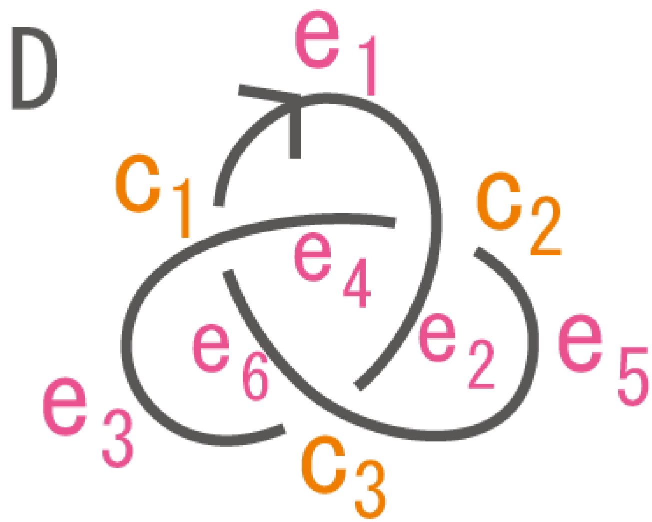
Warping Incidence Matrix



	e_1	e_2	e_3	e_4	e_5	e_6
c_1	0	0	0	1	1	1
c_2	0	1	1	1	0	0
c_3	1	1	0	0	0	1

$$m(D) = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

/

- Switching two rows
- Applying a cyclic permutation on columns

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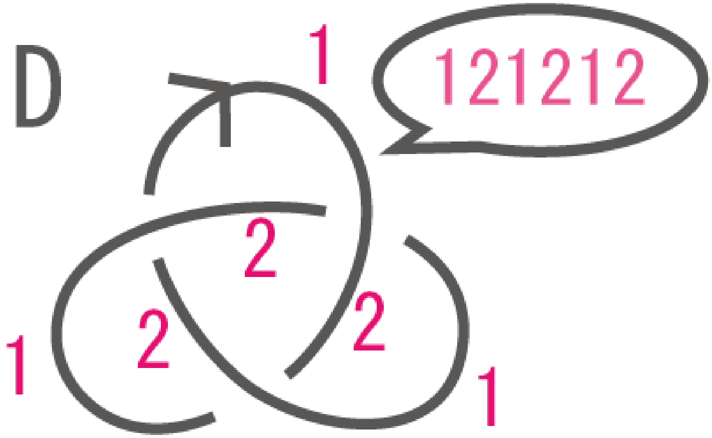
Warping Incidence Matrix



The sum of all the rows of m(D) is the warping degree sequence.

warping incidence matrix

warping degree sequence!

$$(1\ 1\ 1)\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} = (1\ 2\ 1\ 2\ 1\ 2)$$

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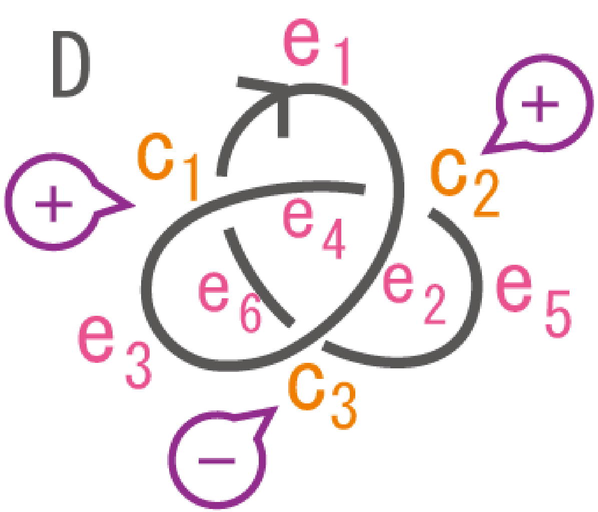
Warping Incidence Matrix

signed warping incidence matrix



	e_1	e_2	e_3	e_4	e_5	e_6
(+) c_1	0	0	0	1	1	1
(+) c_2	0	1	1	1	0	0
(−) c_3	0	0	−1	−1	−1	0

$$m(D) = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 \end{pmatrix}$$

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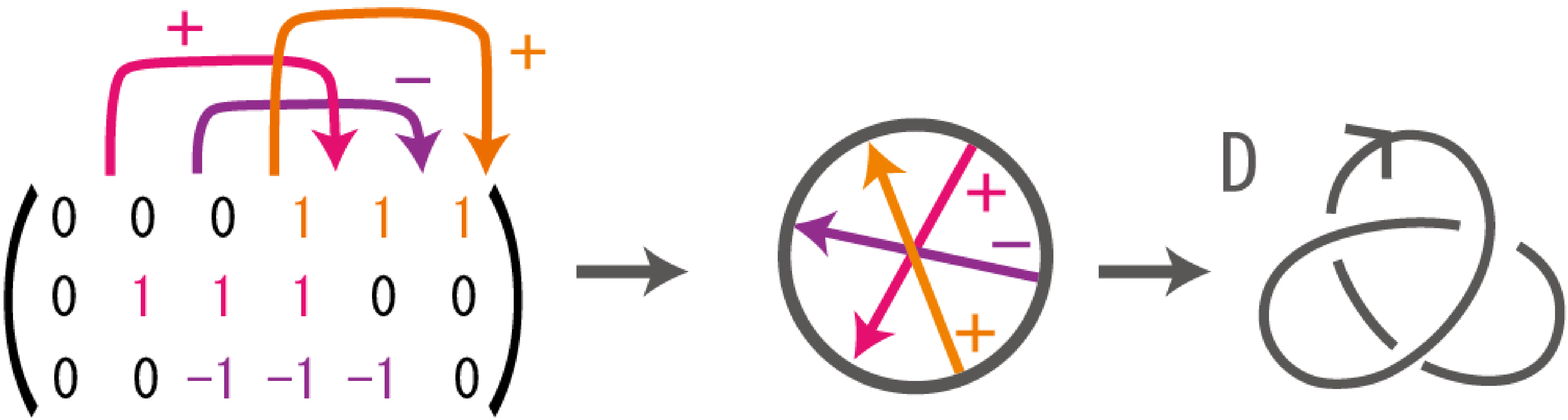
Warping Incidence Matrix

Theorem

Each signed warping incidence matrix represents a knot diagram on S^2 uniquely.



(We can obtain the Gauss diagram from m(D).)

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