ON THE ATTAINABLE DISTRIBUTIONS OF DIFFUSION PROCESSES PERTAINING TO A CHAIN OF DISTRIBUTED SYSTEMS *

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Abstract. In this talk, we consider a diffusion process pertaining to a chain of distributed systems with random perturbations that satisfies a weak Hörmander type condition. In particular, we consider the following stochastic control problem with two objectives. The first one being of a reachability-type that consists of determining the set of attainable distribution laws at a given time starting from an initial distribution law; while the second one involves minimizing the relative entropy of the attainable distribution law with respect to the initial law. Using the logarithmic transformations approach introduced by Fleming (e.g., see [5] and [6]), we provide a sufficient condition on the existence of an optimal admissible control for such a stochastic control problem which is amounted to changing the drift term by a certain perturbation suggested by Jamison in the context of reciprocal processes (e.g., see [8] and [9] (cf. [3])). Moreover, such a perturbation coincides with a minimum energy control among all admissible controls forcing the diffusion process to the desired attainable distribution law starting from the initial law. Finally, using measure transform techniques (e.g., see [7], [4] or [10]), we characterize the most probable path-space for the diffusion process corresponding to such changes in the drift term of the distributed systems.

Key words. Diffusion processes, distributed systems, parabolic equations, relative entropy, stochastic control problem.

AMS subject classifications. 35K10, 35K65, 49J20, 60J60, 93E20, 94A17

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*This work is, in some sense, a continuation of our previous papers [1] and [2].

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