

WORKSHOP ON CALABI–YAU MANIFOLDS
ARITHMETIC, GEOMETRY AND PHYSICS

AUGUST 7–8, 2017

South Building 7, Room 101, Gakushuin University

ABSTRACTS

Carnahan, Scott (Tsukuba University)

51 constructions of the Moonshine module

Abstract: In 1993, Tuite first proposed a natural bijection between algebraic conjugacy classes of non-Fricke elements in the Monster and algebraic conjugacy classes of fixed-point free automorphisms of the Leech lattice, realized by a cyclic orbifold duality in conformal field theories. With recent progress on the rationality and existence of orbifold constructions in the theory of vertex operator algebras, this duality and the bijection are proved. Furthermore, we obtain new moonshine behavior from infinite dimensional Lie algebras obtained by quantizing the conformal field theories.

Hamanaka, Masashi (Nagoya University)

Noncommutative Instantons and Reciprocity

Abstract: In this talk, we discuss $U(N)$ instantons in noncommutative (NC) spaces.

Noncommutative space is a space which coordinate ring is noncommutative. Let x, y be the spacial coordinates. The noncommutativity is expressed by the following commutation relations: $[x, y] = \sqrt{-1}\theta$, where θ is real constant and called the noncommutative parameter. When θ vanishes identically, the coordinate ring is commutative and the underlying space reduces to a commutative one. The commutation relations, like the canonical commutation relations in quantum mechanics, lead to a space-space uncertainty relation. Singularities in commutative space could resolve in noncommutative space thereby. This is one of the prominent features of field theories on noncommutative space and yields various new physical objects such as $U(1)$ instantons.

There are two formalism to describe noncommutative gauge theories: the star-product formalism and the operator formalism. Anti-self-dual (ASD) Yang-Mills equation and the solutions have been studied from the several viewpoints of mathematical physics, particularly, integrable systems, geometry and field theories.

Instantons are finite-action solutions of the ASD Yang-Mills equation and become exact solutions of classical Yang-Mills theories. They can reveal non-perturbative aspects of the quantum theories. Actually, the pathintegrations, formulating the quantum theories, could reduce to finite-dimensional integrations over the instanton moduli spaces. The Atiyah-Drinfeld-Hitchin-Manin (ADHM) construction is a powerful method to obtain the instantons. Furthermore, via the construction, the instanton moduli space is mapped to the set of quadruple matrices which are solutions of the ADHM equation and called the ADHM data. The aforementioned integration, being thereby an integration over the matrices, becomes tractable. To evaluate the integration the use of noncommutative instantons is relevant so that a localization formula can be applied to the integration. In the procedures, various formulas and relations of the ADHM construction are required. Hence it is worthwhile to elucidate the one-to-one correspondence (reciprocity) between moduli spaces of the noncommutative instantons and the ADHM data and to present all the ingredients in the construction explicitly.

We would like to discuss the ADHM construction of noncommutative instantons together with a brief review of commutative ADHM construction from the viewpoint of the Fourier-Mukai-Nahm transformation. We prove the reciprocity in both the star-product formalism and the operator formalism. We reconsider origin of the instanton number by applying an idea of Atiyah and Hori to the noncommutative situation even for the $U(1)$ case. This is based on collaboration with Toshio Nakatsu (Setsunan University).

Hashimoto, Kenji (University of Tokyo)

K3 surfaces and Pell's equation

Abstract: I will discuss automorphisms of K3 surfaces with Picard number 2 which corresponds to solutions of Pell equation. We can apply the theory of binary quadratic forms to study such automorphisms.

Hosono, Shinobu (Gakushuin University)

Gluing monodromy nilpotent cones in mirror symmetry of Calabi-Yau manifolds

Abstract: I will discuss mirror symmetry of Calabi-Yau complete intersections which have birational models and/or birational automorphisms of infinite order. In birational geometry, Kaehler cones of birational models are naturally glued together to a larger cone. Making explicit examples, I will show that, under mirror symmetry, this gluing naturally transformed to the corresponding gluing of the monodromy nilpotent cones for mirror families. This talk is based on works with Hiromichi Takagi.

Kanazawa, Atsushi (Kyoto University)

Weil–Petersson geometry on the space of Bridgeland stability conditions

Abstract: Inspired by mirror symmetry, we investigate some differential geometric aspects of the space of Bridgeland stability conditions on a Calabi–Yau triangulated category. The aim is to develop theory of Weil–Petersson geometry on the stringy Kähler moduli space. I will discuss a few examples and its relation to the Bergman metric. This is based on a joint work with Y.-W. Fan and S.-T. Yau.

Kondo, Shigeyuki (Nagoya University)

Classification of Enriques surfaces with finite automorphism group in characteristic 2

Abstract: We classify supersingular and classical Enriques surfaces with finite automorphism group in characteristic 2 into 8 types according to their dual graphs of all nonsingular rational curves. We give examples of these Enriques surfaces together with their canonical coverings. It follows that the classification of all Enriques surfaces with finite automorphism group in any characteristics has been finished. This is a joint work with Toshiyuki Katsura and Gebhard Martin.

Nakayashiki, Atsushi (Tsuda University)

Degeneration of trigonal curves and corresponding solutions of the KP-hierarchy

Abstract: The KP-hierarchy is one of the most famous soliton equation. It has solutions expressed by the theta functions associated with the Jacobians of non-singular algebraic curves.

We study the limit of theta function solutions when algebraic curves degenerate to singular rational curves. It is known that the so-called soliton solutions correspond to rational curves with only ordinary double points. In this talk we study the limit of the theta function solution when a certain trigonal curve degenerates to a rational curve with only ordinary triple points. We express the corresponding solution in the form of Wronskian of an explicitly defined set of functions.

Ogata, Shoetsu (Tohoku University)

On toric 3-folds defined by quadrics

Abstract: I will show that a projective nonsingular toric 3-fold which is a projective line bundle over a toric surface is defined by polynomials of degree two.

Ohashi, Hisanori (Tokyo University of Science, Noda)

On automorphisms of Enriques surfaces and their entropy

Abstract: We will introduce a new property which is shared by arbitrary Enriques surface automorphisms (over complex numbers). We will give a numerical bound on it, but the existence part remains open at present. The property was inspired by works on the minimum entropy problem on Enriques surfaces, especially the one by Oguiso. Using our property, we will derive a better bound on the minimum entropy. This is a joint work with Y. Matsumoto (Nagoya) and S. Rams (Jagiellonian).

Shimada, Ichiro (Hiroshima University)

Rational double points on complex Enriques surfaces

Abstract: We classify, by some lattice-theoretic invariants, all the possibilities of configurations of rational double points on a surface birational to an Enriques surface.

Yoshikawa, Sho (Gakushuin University)

On modularity of elliptic curves over certain totally real fields

Abstract: Elliptic curves are 1-dimensional Calabi-Yau varieties. This talk is about an aspect of the arithmetic of elliptic curves; especially a natural generalization of the Shimura-Taniyama conjecture.

The original conjecture, which was already solved, states that all elliptic curves over the rationals arise from modular forms. By using "Serre's method", Professor Yui proved an analog to this conjecture for rigid Calabi-Yau threefolds over the rationals. I will talk about another analog; that is, a generalization of the conjecture to all elliptic curves over some totally real fields.

Yui, Noriko (Queen's University)

Supercongruences for rigid hypergeometric Calabi–Yau threefolds

Abstract: We will present two proofs to the supercongruences for the fourteen rigid hypergeometric Calabi–Yau threefolds defined over \mathbf{Q} . The existence of such supercongruences was conjectured (based on numerical evidence) by F. Rodriguez-Villegas in 2003.

This is a joint work with Ling Long, Fang-Ting Tu and Wadim Zudilin.